

# On Field Induced Diaelastic Effect in a Small Josephson Contact

S. Sergeenkov and F.M. Araujo-Moreira

*Grupo de Materiais e Dispositivos, Departamento de Física,  
Universidade Federal de São Carlos, 13565-905 São Carlos, SP, Brazil*

(Dated: February 24, 2013)

## Abstract

An analog of the diaelastic effect is predicted to occur in a small Josephson contact with Josephson vortices manifesting itself as magnetic field induced softening of the contact shear modulus  $C(T, H)$ . In addition to Fraunhofer type *field* oscillations,  $C(T, H)$  is found to exhibit pronounced flux driven *temperature* oscillations near  $T_C$ .

PACS numbers: 74.50.+r, 74.62.Fj, 81.40.Jj

**1. Introduction.** Inspired by new possibilities offered by the cutting-edge nanotechnologies, the experimental and theoretical physics of increasingly sophisticated mesoscopic quantum devices heavily based on Josephson junctions (JJ) and their arrays (JJA) is becoming one of the most exciting and rapidly growing areas of modern science (for the recent reviews, see, e.g., [1–4] and further references therein ). In particular, a remarkable increase of the measurements technique resolution made it possible to experimentally detect such interesting phenomena as flux avalanches [5], geometric quantization [6], flux driven oscillations of heat capacity [7], reentrant-like behavior [8], manifestation of  $\pi$ -contacts [9],  $R$ - $C$  crossover [10], unusually strong coherent response [11], Josephson analog of the fish-tail effect [12], geometric resonance and field induced Kosterlitz-Thouless transition [13]. Among the numerous theoretical predictions (still awaiting their experimental verification) one could mention electro- and magnetostriction [14], field induced polarization effects [15], analog of magnetoelectric effect [16], nonlinear Seebeck effect and thermal conductivity [17], stress induced effects [18], chemomagnetism [19], magnetoinductance effects [20], implications of dipolar interactions for wireless connection between Josephson qubits [21] and for weakening of the Coulomb blockade [22], proximity-induced superconductivity in graphene [23] and anomalous Josephson current in topological insulators [24].

Turning to the subject of this Letter, let us recall that when an elastic solid contains a region with compressibility different from the bulk one, the applied stress  $\sigma$  induces a spatially inhomogeneous strain field  $\epsilon$  around this region, which results in the softening of its shear modulus  $C$ . This phenomenon, known as diaelastic effect (DE), usually occurs in materials with pronounced defect structure [25]. By association, Josephson vortices can be considered as defects related inclusions within tunneling contacts. Therefore, one could expect an appearance of magnetic field induced analog of DE in Josephson structures as well. By introducing an elastic response of JJ to an effective stress field, in what follows we shall discuss a possible manifestation of this novel interesting effect in a small contact under an applied magnetic field.

**2. Model.** The temperature and field dependence of the elastic shear modulus  $C(T, H)$  of the Josephson structure can be defined as follows (Cf. Ref. [18]):

$$\frac{1}{C(T, H)} = \left[ \frac{d\epsilon(T, H, \sigma)}{d\sigma} \right]_{\sigma=0} \quad (1)$$

where  $\sigma$  is an applied stress and strain field  $\epsilon$  in the contact area is related to the stress

dependent Josephson critical current  $I_C$  as follows ( $V$  is the volume of the sample) [18]:

$$\epsilon(T, H, \sigma) = \left( \frac{\Phi_0}{2\pi V} \right) \frac{dI_C(T, H, \sigma)}{d\sigma} \quad (2)$$

For simplicity and to avoid self-field effects, in what follows we consider a small Josephson contact of length  $w < \lambda_J$  ( $\lambda_J = \sqrt{\Phi_0/\mu_0 d j_c}$  is the Josephson penetration depth) placed in a strong enough magnetic field (which is applied normally to the contact area) such that  $H > \Phi_0/2\pi\lambda_J d$ , where  $d = 2\lambda_L + t$ ,  $\lambda_L$  is the London penetration depth, and  $t$  is an insulator thickness.

Recall that the critical current of such a contact in applied magnetic field is governed by a Fraunhofer-like dependence [26]:

$$I_C(T, H, \sigma) = I_C(T, 0, \sigma) \left[ \frac{\sin \varphi(T, H, \sigma)}{\varphi(T, H, \sigma)} \right] \quad (3)$$

where  $\varphi(T, H, \sigma) = \pi\Phi(T, H, \sigma)/\Phi_0$  with  $\Phi(T, H, \sigma) = Hwd(T, \sigma)$  being the temperature and stress dependent flux through the contact area, and  $I_C(T, 0, \sigma) \propto e^{-t/\xi}$  is the stress dependent zero-field Josephson critical current with  $\xi$  being a characteristic (decaying) length and  $t(\sigma)$  the stress dependent thickness of the insulating layer (see below).

Notice that in non-zero applied magnetic field  $H$ , there are two stress-induced contributions to the critical current  $I_C$ , both related to decreasing of the insulator thickness under pressure. First of all, it was experimentally observed [27] that the tunneling dominated critical current of granular superconductors exponentially increases under compressive stress  $\sigma$ , viz.  $I_C \propto e^{\kappa\sigma}$ . More specifically, the critical current at  $\sigma = 9kbar$  was found to be three times higher its value at  $\sigma = 1.5kbar$ , clearly indicating a weak-links-mediated origin of the phenomenon. Hence, for small enough  $\sigma$  we can safely assume that [18]  $t(\sigma) \simeq t(0)(1 - \beta\sigma)$ . As a result, we have two stress-induced effects in Josephson contacts: (a) amplitude modulation leading to the explicit stress dependence of the zero-field current

$$I_C(T, 0, \sigma) = I_C(T, 0, 0)e^{\gamma\sigma} \quad (4)$$

with  $\gamma = \beta t(0)/\xi$ , and (b) phase modulation leading to the explicit stress dependence of the flux

$$\Phi(T, H, \sigma) = Hwd(T, \sigma) \quad (5)$$

with

$$d(T, \sigma) = 2\lambda_L(T) + t(0)(1 - \beta\sigma) \quad (6)$$

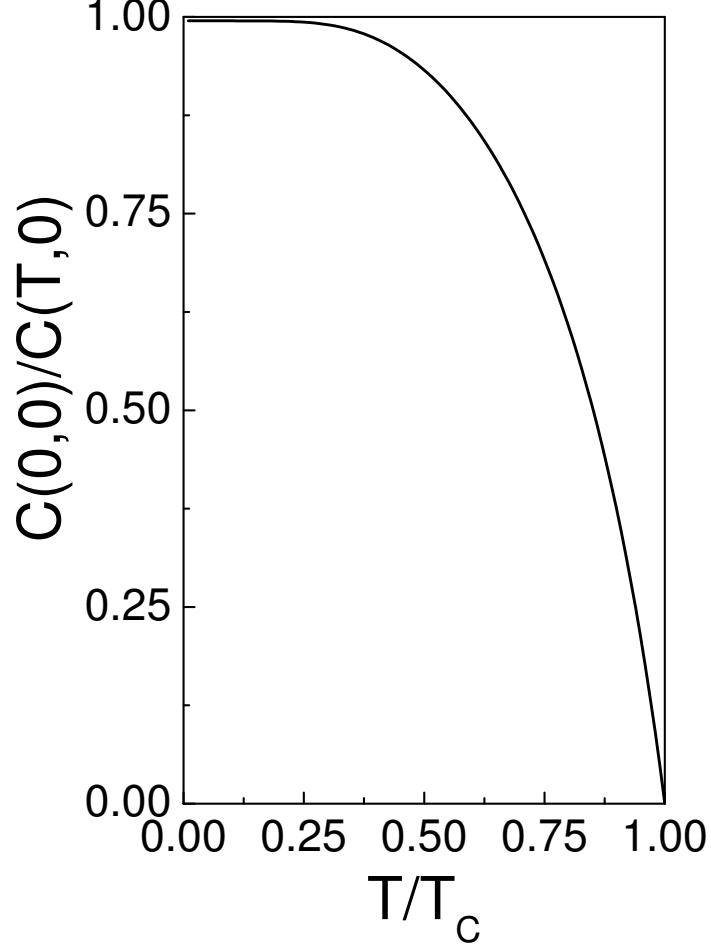


FIG. 1: Temperature dependence of the normalized inverse shear modulus  $C(0,0)/C(T,0)$  of a single short contact in zero magnetic field according to Eqs.(1)-(12).

Finally, in view of Eqs.(1)-(6), the temperature and field dependence of the small single junction shear modulus  $C(T, H)$  reads:

$$\frac{1}{C(T, H)} = \frac{1}{C(T, 0)} \left[ F(T, H) - \frac{\xi}{d(T, 0)} \frac{dF(T, H)}{d \log H} \right] \quad (7)$$

where

$$F(T, H) = \left[ \frac{\sin \varphi}{\varphi} + \frac{\xi}{d(T, 0)} \left( \frac{\sin \varphi}{\varphi} - \cos \varphi \right) \right] \quad (8)$$

with

$$\varphi(T, H) = \frac{\pi \Phi(T, H, 0)}{\Phi_0} = \frac{H}{H_0(T)} \quad (9)$$

and

$$\frac{1}{C(T, 0)} = \left( \frac{\Phi_0 \gamma^2}{2\pi V} \right) I_C(T) \quad (10)$$

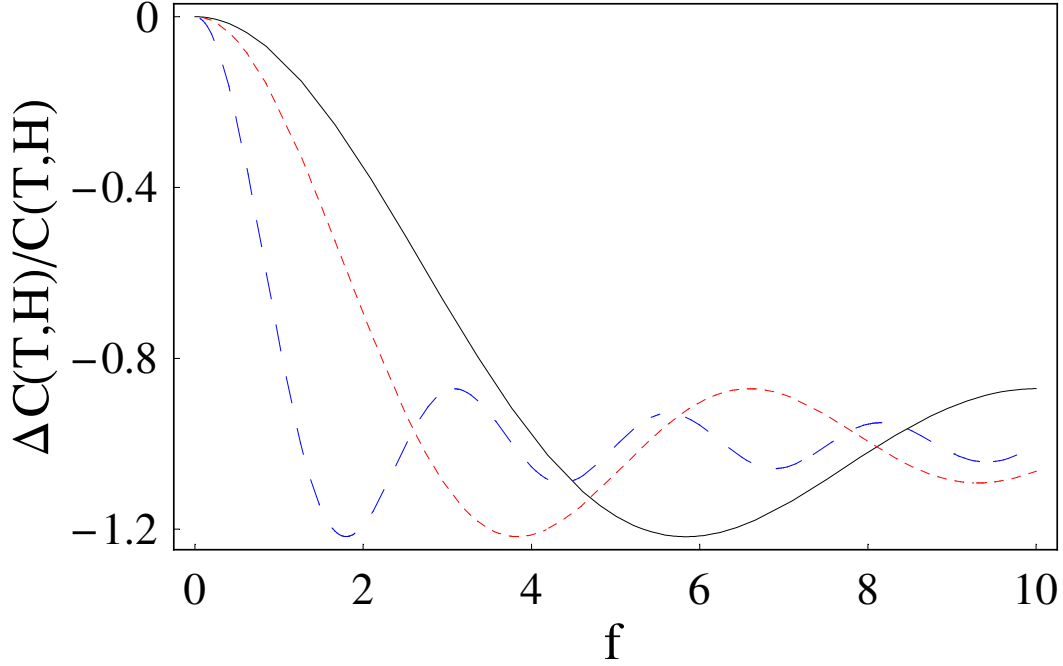


FIG. 2: The field dependence of the diaelastic effect  $\Delta C(T, H)/C(T, H)$  for different temperatures:  $T = 0$  (solid line),  $T = 0.5T_C$  (dotted line), and  $T = 0.9T_C$  (dashed line).

Here,  $H_0(T) = \Phi_0/\pi wd(T, 0)$  with  $d(T, 0) = 2\lambda_L(T) + t(0)$ , and for convenience we used a simplified definition  $I_C(T, 0, 0) \equiv I_C(T)$  for zero-field and zero-stress critical current.

For the explicit temperature dependence of  $I_C(T)$  we use the analytical approximation of the BCS gap parameter (valid for all temperatures) [17],  $\Delta(T) = \Delta(0) \tanh\left(2.2\sqrt{\frac{T_C - T}{T}}\right)$  with  $\Delta(0) = 1.76k_B T_C$  which governs the temperature dependence of the Josephson critical current

$$I_C(T) = I_C(0) \left[ \frac{\Delta(T)}{\Delta(0)} \right] \tanh \left[ \frac{\Delta(T)}{2k_B T} \right] \quad (11)$$

while the temperature dependence of the London penetration depth is governed by the two-fluid model [28]:

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - (T/T_C)^4}} \quad (12)$$

**3. Results and Discussion.** Fig. 1 presents the temperature behavior of the contact area shear modulus  $C(T, 0)$  (with  $t(0)/\xi = 1$ ,  $\xi/\lambda_L(0) = 0.02$  and  $\beta = 0.1$ ) in zero applied magnetic field. Notice that  $C(T, 0)$  is positive for all temperatures. The considered here field

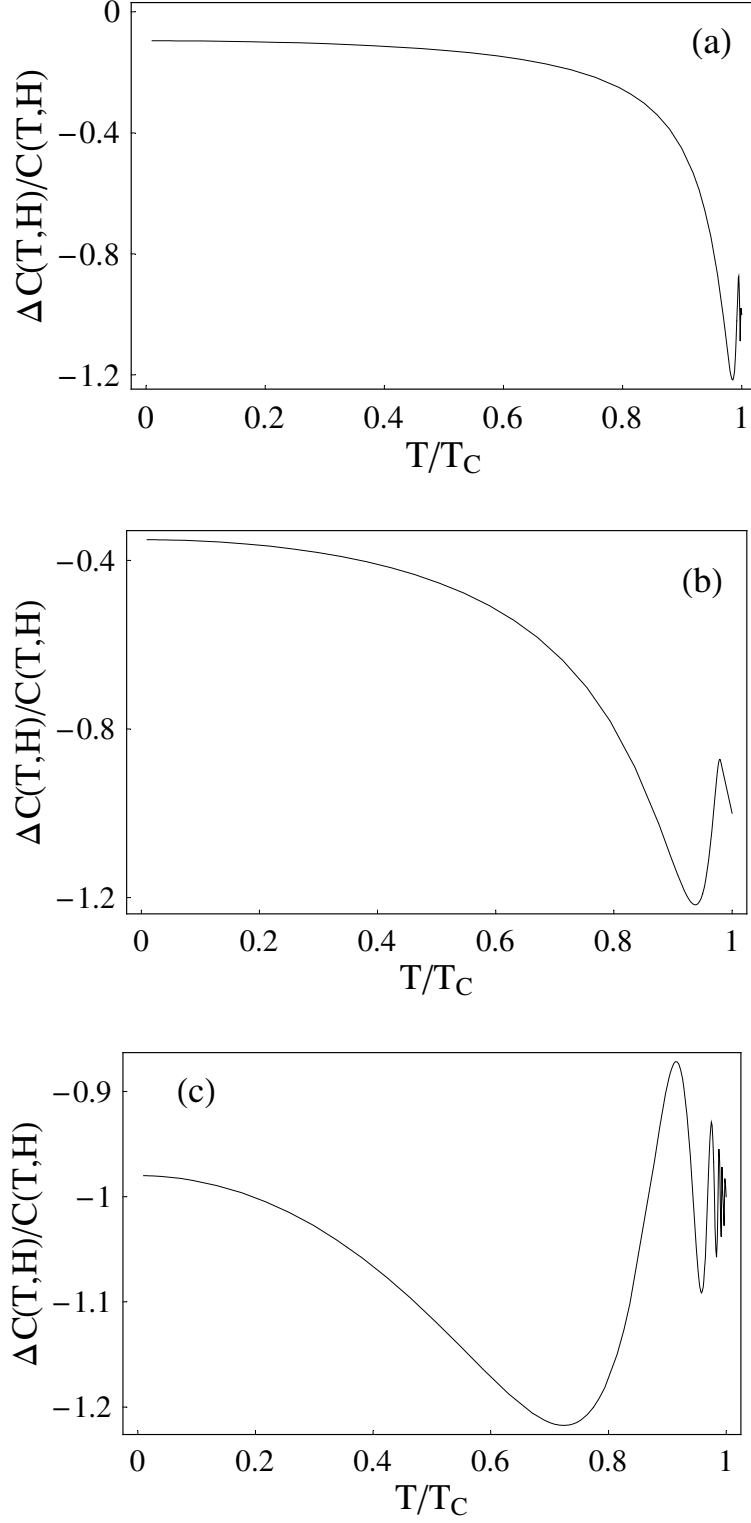


FIG. 3: Temperature dependence of the diaelastic effect  $\Delta C(T,H)/C(T,H)$  for different values of the frustration parameter  $f$ : (a)  $f = 1$ , (b)  $f = 3$ , and (c)  $f = 5$ .

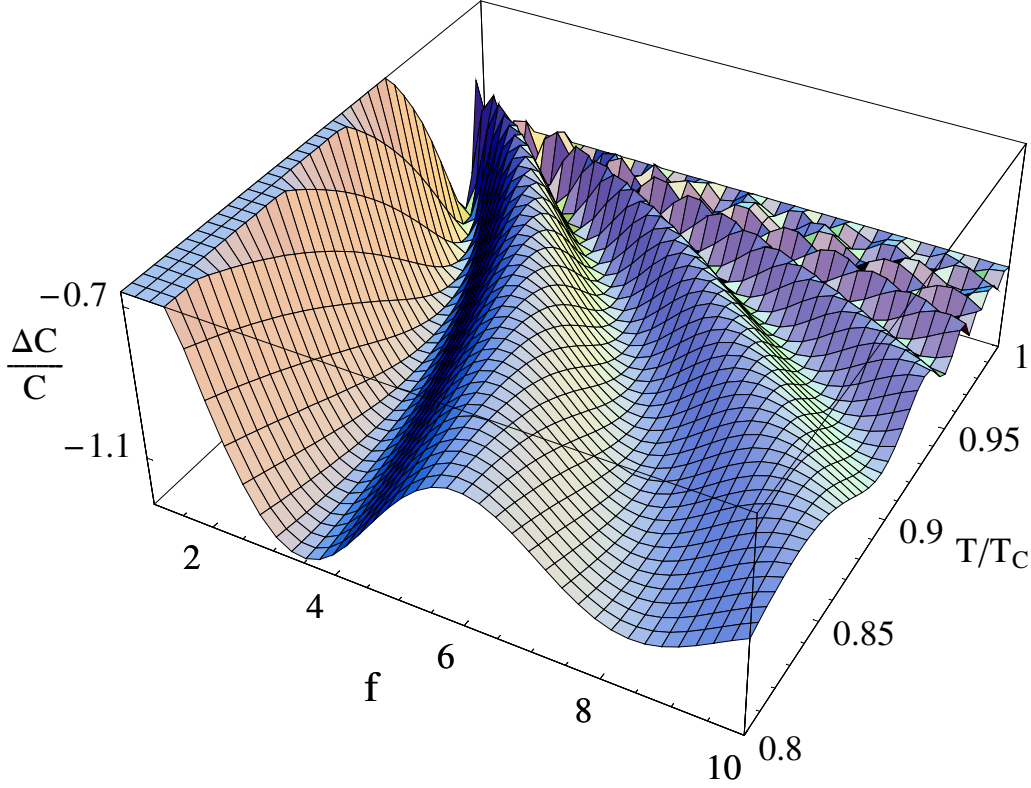


FIG. 4: 3D flux-temperature profile of the diaelastic effect  $\Delta C(T, H)/C(T, H)$ .

induced analog of the diaelastic effect means *softening* of the contact area shear modulus under the influence of the applied magnetic field with  $\Delta C(T, H) = C(T, H) - C(T, 0) < 0$ . Fig. 2 demonstrates this predicted behavior showing the field dependence of the DE  $\Delta C(T, H)/C(T, H)$  for different temperatures. As it would be expected from the very structure of Eqs.(1)-(9), the DE of a single contact exhibits *field* oscillations imposed by the Fraunhofer dependence of the critical current  $I_C$ . Even more interesting is its temperature dependence. Indeed, according to Fig. 3, depicting the temperature dependence of the DE for different values of the frustration parameter  $f = H/H_0(0)$ , we see characteristic flux driven *temperature* oscillations of  $\Delta C(T, H)$  near  $T_C$ . These oscillations are governed by the temperature dependence of the London penetration depth  $\lambda_L(T)$ , which controls the number of fluxons entering Josephson contact at a given temperature via the characteristic field  $H_0 \simeq \Phi_0/2\pi\lambda_L w$ . A more spectacular view of the double temperature-flux oscillations of the DE can be seen through its 3D image, presented in Fig. 4. The predicted here effect

should manifest itself not only in single JJs and highly ordered JJAs but also in granular superconductors (described as disordered JJAs) including the so-called nanogranular (or intrinsically granular) superconductors [2, 4]. In the latter case, however, the influence of Abrikosov vortices on weak-link mediated DE should be taken into account due to high values of the characteristic field  $H_0$  reaching a few Tesla for contact size  $w$  of a few nanometers.

In summary, by considering an elastic response of a small Josephson contact to an effective applied stress field, an analog of the so-called diaelastic effect was predicted to occur in such a contact manifesting itself as a magnetic field induced softening of its shear modulus with pronounced field and temperature oscillations.

This work has been financially supported by the Brazilian agencies CAPES, CNPq, and FAPESP.

- 
- [1] *New Directions in Mesoscopic Physics: Towards Nanoscience*, Eds. R. Fazio, V.F. Gantmakher, and Y. Imry (Kluwer Academic Publishers, Dordrecht, 2003).
  - [2] I.S. Beloborodov, A.V. Lopatin, V.M. Vinokur et al., *Rev. Mod. Phys.* **79**, 469 (2007).
  - [3] *New Developments in Josephson Junctions Research*, Ed. S. Sergeenkov (Transworld Research Network Publisher, Kerala, 2010).
  - [4] S. Sergeenkov, in *Oxford Handbook of Nanoscience and Technology: Frontiers and Advances*, Eds. A. Narlikar and Y.Y. Fu (Oxford University Press, New York, 2010), vol. I, pp. 703–731.
  - [5] S.M. Ishikae, E.V. Matizen, V.V. Ryazanov et al., *JETP Lett.* **72**, 26 (2000); E.V. Matizen, S.M. Ishikae and K.R. Zhdanov, *J. Mol. Liquids* **148**, 29 (2009); E.V. Matizen, V.G. Martynets and P.P. Bezverkhii, *JETP* **111**, 309 (2010).
  - [6] S. Sergeenkov and F.M. Araujo-Moreira, *JETP Lett.* **80**, 580 (2004).
  - [7] O. Bourgeois, S. E. Skipetrov, F. Ong et al., *Phys. Rev. Lett.* **94**, 057007 (2005).
  - [8] F.M. Araujo-Moreira, W. Maluf and S. Sergeenkov, *Eur. Phys. J. B* **44**, 33 (2005); F.M. Araujo-Moreira and S. Sergeenkov, *Supercond. Sci. and Technol.* **21**, 045002 (2008).
  - [9] V.V. Ryazanov, V.A. Oboznov, A.S. Prokofiev et al., *J. Low Temp. Phys.* **136**, 385 (2004); V.A. Oboznov, V.V. Bol’ginov, A.K. Feofanov et al., *Phys. Rev. Lett.* **96**, 197003 (2006); V.A.G. Rivera, S. Sergeenkov, E. Marega et al., *Phys. Lett. A* **374**, 376 (2009); M.I. Khabipov, D.V. Balashov, F. Maibaum et al., *Supercond. Sci. and Technol.* **23**, 45032 (2010).



- [10] S. Sergeenkov, V.A.G. Rivera, E. Marega et al., J. Appl. Phys. **107**, 096102 (2010).
- [11] V.A.G. Rivera, S. Sergeenkov, C. Stari et al., JETP Lett. **90**, 365 (2009).
- [12] S. Sergeenkov, L.Jr. Cichetto, V.A.G. Rivera et al., JETP Lett. **91**, 32 (2010).
- [13] V.A.G. Rivera, S. Sergeenkov, E. Marega et al., Physica C **470**, 1946 (2010); S. Sergeenkov, V.A.G. Rivera, E. Marega et al., Solid State Commun. **151**, 892 (2011).
- [14] S. Sergeenkov, Solid State Commun. **106**, 451 (1998); S. Sergeenkov and M. Ausloos, Phys. Rev. B **48**, 604 (1993).
- [15] S. Sergeenkov, J. de Phys. I (France) **7**, 1175 (1997); S. Sergeenkov, JETP **101**, 919 (2005).
- [16] S. Sergeenkov and J. José, Europhys. Lett. **43**, 469 (1998).
- [17] S. Sergeenkov, JETP Lett. **67**, 680 (1998); S. Sergeenkov, JETP Lett. **76**, 170 (2002).
- [18] S. Sergeenkov, J. of Phys.: Cond. Matter **10**, L265 (1998); S. Sergeenkov, JETP Lett. **70**, 36 (1999); S. Sergeenkov, JETP Lett. **75**, 317 (2002); S. Sergeenkov, G. Rotoli, G. Filatrella et al., Phys. Rev. B **75**, 014506 (2007).
- [19] S. Sergeenkov, JETP Lett. **77**, 94 (2003); S. Sergeenkov, J. Appl. Phys. **102**, 066104 (2007).
- [20] S.E. Korshunov, Phys. Rev. B **68**, 094512 (2003); S. Sergeenkov, Phys. Lett. A **372**, 2917 (2008).
- [21] S. Sergeenkov and G. Rotoli, Phys. Rev. A **79**, 044301 (2009).
- [22] S. Sergeenkov, AIP Advances **1**, 022122 (2011).
- [23] M.V. Feigel'man, M.A. Skvortsov and K.S. Tikhonov, Solid State Commun. **149**, 1101 (2009).
- [24] P.A. Ioselevich and M.V. Feigel'man, Phys. Rev. Lett. **106**, 77003 (2011).
- [25] B. Igarashi, E.C. Johnson and A.V. Granato, Phys. Rev. B **48**, 2909 (1993); J. Kikas and K. Leiger, J. Chem. Phys. **104**, 5384 (1996).
- [26] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (A Wiley-Interscience Publisher, New York, 1982).
- [27] A.I. D'yachenko, V.Y. Tarenkov, A.V. Abalioshev, R.V. Luticiv, Y.N. Myasoedov, and Y.V. Boiko, Physica C **251**, 207 (1995).
- [28] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, Singapore, 1997).